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Acousto-optic modulation in magnetised diffusive semiconductors

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Abstract. The modulation of an intense electromagnetic beam induced by the acousto-optic (AO) effect has been analysed in a transversely magnetised semiconductor-plasma medium. The effect of carrier diffusion on the threshold field and gain profile of the modulated wave has been extremely investigated using coupled mode theory. The origin of the AO interaction is assumed to lie in the induced nonlinear diffusion current density of the medium. By considering the modulation process as a four wave parametric interaction an expression for effective third-order AO susceptibility describing the phenomena has been deduced. The modulation is greatly modified by propagation characteristics such as dispersion and diffraction due to dielectric relaxation of the acoustic mode. The threshold pump field and the steady state growth rates are estimated from the effective third-order polarisation in the plasma medium. Analytical estimation reveals that in the presence of enhanced diffusion due to excess charge carriers the modulated beam can be effectively amplified in a dispersionless acoustic wave regime. The presence of an external dc magnetic field is found to be favourable for the onset of diffusion induced modulational amplification of the modulated wave in heavily doped regime.

PACS. 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 78.20.Hp Piezo-, elasto-, and acoustooptical effects; photoacoustic effects – 42.65.An Optical susceptibility, hyperpolarizability – 42.65.Ky Harmonic generation, frequency conversion

1 Introduction

In the present paper, by considering that the origin of modulational interaction lies in the third-order optical susceptibility $\chi^{(3)}$ arising from the nonlinear diffusion current density and using the coupled mode theory of plasmas, an analytical investigation of frequency modulational interaction between copropagating laser beams and internally generated acoustic mode is presented. We have also studied the steady state amplification characteristics of modulated waves arising from the nonlinearity of the current density in a transversely magnetised diffusive centrosymmetric semiconductor.

The interaction of high power lasers with semiconductor plasma has been playing a prominent role in diverse areas of scientific research for several decades [1–3]. It has immense applications in processing of materials and fabrication of devices [4,5]. Semiconductors also provide a compact and less expensive medium to model nonlinear phenomena encountered in laser produced plasmas. There exists a number of nonlinear interactions which can be classified as modulational interaction. The resulting amplification of decay channels by modulational interactions are generally referred to as an instability of wave propagating in nonlinear dispersive medium such that the steady state becomes unstable and evolves into a temporally modulated state [6].

The concept of transverse modulational instability originates from a space-time analogy that exists when the dispersion is replaced by diffraction [7]. The field induced change in the refractive index due to a change in the local optical characteristics of semiconducting medium leads to modulational instability, nonlinear focusing, or filamentation of propagating beams. Morever, electro-optic and acousto-optic (AO) effects afford a convenient and widely used means of controlling intensity and/or phase of the propagating radiation [1,2]. This modulation is used in ever-expending number of applications including the impression of information onto optical pulses, mode-locking, and optical beam deflection [3,8]. The modulation of electromagnetic beams by surface-acoustic waves is also a very active field of interest due to their applicability in the field of communication devices [9].

It is known that the acoustic wave diffracts the light-beam within the active medium and provides an effective mechanism for nonlinear optical response in AO devices. Specifically, the photo elastic effect in a medium causes a variation in the medium's refractive index which is proportional to an acoustic perturbation and implies the existence of a corresponding electrostrictive effect. It induces an acoustic response in the medium that is a spatially varying quadratic function of the local electric

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field. In the Bragg regime, high diffraction efficiencies of an AO device due to induced electrostrictive effects was predicted by Yeh and Khoshnevisan [10]. Vachss and Mc Michael [11] have demonstrated this acousto-optic gain in TeO_2 crystals. The optically induced strain wave also depend on the physical scattering properties of the material. This effect permits optical modification of the local acoustic waves [12]. Using Fourier transform approach Banerjee [13] has recently reported the acousto-optic interaction between arbitrary light and sound profiles.

The nonlinear interaction of electromagnetic waves in bulk and thin dielectrics has been studied in recent years by a number of workers [14–19]. AO interaction in dielectrics and semiconductors is playing an important role in optical modulation and beam steering [4,5]. However, in integrated optoelectronic device application, the AO modulation process becomes a serious limitation due to high acoustic power requirements. The most direct approach to this problem is to tailor new materials with more desirable AO properties.

In most cases of investigation of nonlinear optical interaction, the nonlocal effects such as diffusion of the excitation carrier density that is expected to be responsible for the nonlinear refractive index change has been ignored. It is found that increased diffusion makes light transmission more difficult and tends to wash out the local equilibria of the equivalent potential representing unstable or stable TE nonlinear surface waves [14]. The high mobility charge carriers makes diffusion effects even more relevant in semiconductor technology as they (charge carriers) travel significant distances before recombining. Therefore inclusion of carrier diffusion in theoretical studies of nonlinear wave-wave interactions seems to be very important from the fundamental as well as application view points and thus attracted many workers in the last decades [12,14,16,20]. The investigation of reflectance and transmittance of a Gaussian beam incident on an interface separating a linear and nonlinear diffusive media have further stimulated research in this direction [14,16]. The diffusion is expected to alter the third-order optical susceptibility $\chi^{(3)}$ and hence significantly changes dispersion and transmission of the incident radiation in the medium [16]. However, it appears from the available literature that no attempt has so far been made on the important role of diffusion on frequency modulation and related phenomena in semiconductor plasmas.

Motivated by the above discussion, in the present article we have presented an analytical study on induced acousto-optic modulation of an intense electromagnetic beam in a strain dependent semiconductor plasma medium in the presence of excess charge carriers. The effect of diffusion-induced current density on the nonlinear interactions of a laser beam adds new dimensions to the analysis presented in an n-type semiconductor [21]. The intense pump beam electrostrictively generates an acoustic wave within the semiconductor medium that induces an interaction between the free carriers through electron plasma wave and the acoustic phonons through material vibrations. This interaction induces a strong space-charge field that modulates the pump beam. Thus the optical and acoustic waves present in an acousto-optic modulator can be strongly amplified through nonlinear optical pumping. The high degree of amplification results from an acoustic gain mechanism that counteracts the usual attenuation of sound wave propagating in an acousto-optic medium [11].

The analysis is based on coupled mode theory [22] for investigating the modulational instability due to parametric four-wave mixing process. The acousto-optic field couples with the modulated signal in the presence of a strain and amplifies it under appropriate phase-matching conditions. The parametric process is characterised by the effective third-order optical susceptibility induced due to diffusion current density in a centrosymmetric semiconductor plasma medium. In electro-optic Kerr effect, the nonlinearity arises from the interaction of optical field with bound electrons, therefore the optical field affects the nonlinear polarisation locally. In the present case, that can be termed as electrostrictive Kerr effect, the nonlinearity is solely due to the diffusion of free carriers and the effect is non-local. The article is organised in the following manner. In Section 2 the effective AO (third-order) susceptibility describing the four wave interaction has been deduced from single-component fluid model of plasma and Maxwell's equations. A linear stability analysis of the growth rate of the modulated signal is presented. The threshold pump intensity required to incite the transverse modulational amplification has also been derived. Section 3 deals with the numerical estimations of the threshold intensity and gain of the modulated signal wave and their dependence on the external parameters. This section also enlists the important conclusions that can be drawn from the present analytical study.

2 Theoretical formulation

An active AO semiconductor crystal is considered to be illuminated by a uniform and homogeneous optical pump beam

$$\mathbf{E}_0 = \mathbf{x}E_0 \exp(-\mathrm{i}\omega_0 t), \tag{1}$$

which copropagates with a parametrically generated acoustic wave within the medium. The medium is immersed in a transverse dc magnetic field $\mathbf{B}_\mathrm{s} = \mathbf{z}B_\mathrm{s}$. Due to medium's photoelastic response, these acoustic grating results in a proportional refractive index variation. The incident optical field will be diffracted by this grating to produce an additional field within the medium. The diffracted beam is either frequency upshifted (anti-stokes mode) or down shifted (stokes mode) depending on the orientation of the incident wave. In the presence of strain the stokes and anti-stokes mode can be coupled over a long interaction path. This coupled wave propagates as a solitary wave form in the dispersionless regime of the acoustic wave and can be amplified under appropriate phase matching conditions. In equation (1), under the dipole approximation the incident pump beam is assumed to be spatially uniform when the excited wave haves wavelengths which one very small as compared to the scale length of the pump field variation [23] (*i.e.* $\mathbf{k}_0 \ll \mathbf{k}$ so that $\mathbf{k}_0$ may be safely neglected).

Here the use of hydrodynamical model of plasma for centrosymmetric semiconductor medium at 77 K (liquid nitrogen temperature) enables us to replace the streaming electrons with an electron fluid described by a few macroscopic parameters like average velocity, average carrier density etc. This replacement simplifies our analysis, without any loss of significant informations. However, it restricts our analysis to be valid only in the limit ($k_\mathrm{a} l \ll 1$; k_a the acoustic wave number, and l the carrier mean free path).

The basic equations governing the said modulational interactions are

$$\frac{\partial \mathbf{v}_0}{\partial t} + \nu \mathbf{v}_0 = \frac{e}{m}(\mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_\mathrm{s}) = \frac{e}{m}\mathbf{E}_\mathrm{eff}, \quad (2)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \nu \mathbf{v}_1 + \left(\mathbf{v}_0 \cdot \frac{\partial}{\partial x}\right)\mathbf{v}_1 = \frac{e}{m}(\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_\mathrm{s}), \quad (3)$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + \nu_0 \frac{\partial n_1}{\partial x} - D\frac{\partial^2 n_1}{\partial x^2} = 0, \quad (4)$$

in which diffusion coefficient

$$D = \frac{k_\mathrm{B} T}{e}\mu. \quad (5)$$

The subscripts 0 and 1 correspond to the physical quantities related to pump and signal modes, respectively.

Equations (2, 3) are the momentum transfer equations for the pump and the product waves, respectively in which $\mathbf{v}_0$ and $\mathbf{v}_1$ are the oscillatory fluid velocities under the influence of the respective fields. ν and m represent the phenomenological momentum transfer collision frequency and effective mass of electrons. Under the assumption $\omega_\mathrm{p} \sim \omega_0$ the contribution of pump magnetic field is neglected. Equation (3) represents the continuity equation in which n_0 and n_1 are the equilibrium and the perturbed carrier concentrations, respectively. In equation (5) $\mu\ (= e/m\nu)$ is the electron mobility k_B is the Boltzmann's constant and T the temperature in K. The basic nonlinearity induced in the motion of the charge carriers is due to the convective derivative $(\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}$ and Lorentz force $e(\mathbf{v} \times \mathbf{B})$ which are the functions of the total intensity of illumination $\mathbf{v}_{0,1}$.

In the multimode theory of modulational interaction the pump beam generates an acoustic perturbation due to the lattice vibrations at the phonon mode frequencies within the semiconductor. These lattice vibrations lead to an electron-density perturbation which couples nonlinearly with the pump wave and drives the acoustic waves at modulated frequencies. The equation of motion of the acoustic wave in a centrosymmetric electrostrictive medium is given by

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{c}{\rho}\frac{\partial^2 \mathbf{u}}{\partial x^2} + 2\gamma \frac{\partial \mathbf{u}}{\partial t} = \frac{1}{2\rho}\varepsilon(\eta^2 - 1)\frac{\partial}{\partial x}(\mathbf{E}_\mathrm{eff} \cdot \mathbf{E}_1^*), \quad (6)$$

where $\mathbf{u}$ is the lattice displacement under the influence of the interfering electromagnetic fields represented by the generalized force on the right hand side of equation (6), ρ is the mass density of the crystal, c the elastic constant, η the linear refractive index, γ the damping constant and ε the permittivity of the crystal. The acoustic field thus generated is also assumed to have a plane wave variation $\exp[\mathrm{i}(k_\mathrm{a} x - \omega_\mathrm{a} t)]$.

The migration of charge carriers *via* diffusion produces a charge separation that leads to a strong space-charge field. This space-charge can thus be obtained from the continuity equation [Eq. (4)] and the Poisson's equation for superposition of Coulomb fields arising from the excess charge density n_1 and free or equilibrium density n_0 [Eq. (7)], as

$$\frac{\partial \mathbf{E}_1}{\partial x} = \frac{n_1 e}{\varepsilon} + \frac{(\eta^2 - 1)}{\varepsilon_1}\mathbf{E}_\mathrm{eff}\frac{\partial^2 u^*}{\partial x^2} \cdot \quad (7)$$

The induced current density $\mathbf{j}(x, t)$ is assumed to consist drift and diffusion terms near thermal equilibrium at 77 K whose x-component may be represented in the form

$$\mathbf{j} = e\mu n \mathbf{E} - eD\frac{\partial \mathbf{n}}{\partial x} \cdot \quad (8)$$

The carrier density perturbation induced by the strong pump beam is associated with the phonon-mode and varies at the acoustic frequency. The pump beam is thus phase modulated by the density perturbations to produced enforced disturbances at the upper $(\omega_\mathrm{a} + \omega_0)$ and lower $(\omega_\mathrm{a} - \omega_0)$ side band frequencies. The higher order frequency components are filtered out by assuming a long-interaction path. The modulation process under consideration must also fulfill the phase matching conditions $\mathbf{k}_0 = \mathbf{k}_1 \pm \mathbf{k}_\mathrm{a}$; $\omega_0 = \omega_1 \pm \omega_\mathrm{a}$ under spatially uniform laser irradiation. The equation for carrier density fluctuation of the coupled electron-plasma wave in a magnetised n-type semiconductor, is obtained by employing equations (1–8) and the linearised perturbation theory as

$$\frac{\partial^2 n_1(\omega_\pm, k_\pm)}{\partial t^2} + \nu \frac{\partial n_1(\omega_\pm, k_\pm)}{\partial t} + \overline{\omega}_\mathrm{p}^2 n_1(\omega_\pm, k_\pm) - \nu D \frac{\partial^2 n_1(\omega_\pm, k_\pm)}{\partial x^2} - \frac{n_0 e k_\mathrm{a}^2(\eta^2 - 1)}{m\varepsilon_1}E_\mathrm{eff}u^* = -\overline{E}\frac{\partial n_1(\omega_\pm, k_\pm)}{\partial x}, \quad (9)$$

in which $\overline{E} = (e/m)E_\mathrm{eff}$ and $\overline{\omega}_\mathrm{p}^2 = \omega_\mathrm{p}^2(\nu^2/(\nu^2 + \omega_\mathrm{c}^2)$. Here $\omega_\mathrm{p}\ [= (n_0 e^2/m\varepsilon)^{\frac{1}{2}}]$ is the plasma frequency and ω_c $(= eB_0/m)$ is the cyclotron frequency of the carriers. The corresponding density modulation oscillating at the upper and lower side band frequencies can be represent by the expression

$$n_1(\omega_\pm, k_\pm) = \frac{n_0 e k_\mathrm{a}^2(\eta^2 - 1)E_\mathrm{eff}u^*}{m\varepsilon_1} \times \left[\overline{\omega}_\mathrm{p}^2 + \nu D k_\pm^2 - \omega_\pm^2 - \mathrm{i}\nu\omega_\pm + \mathrm{i}k_\pm \overline{E}\right]^{-1} \quad (10)$$

where

$$u^* = \frac{-\mathrm{i}k_\mathrm{a}\varepsilon(\eta^2 - 1)E_\mathrm{eff}^* E_1}{2\rho\left[\omega_\mathrm{a}^2 - k_\mathrm{a}^2\nu_\mathrm{a}^2 - 2\mathrm{i}\gamma\omega_\mathrm{a}\right]} \quad (11)$$

is obtained from equation (6) substitution of equation (11) in (10) yields,

$$n_1(\omega_\pm, k_\pm) = \frac{-\mathrm{i}n_0\varepsilon_0 e k_\mathrm{a}^3(\eta^2-1)^2\left|E_\mathrm{eff}\right|^2 E_1}{2\rho m\left[\omega_\mathrm{a}^2 - k_\mathrm{a}^2\nu_\mathrm{a}^2 - 2\mathrm{i}\gamma\omega_\mathrm{a}\right]} \times\left[\overline{\varpi}_\mathrm{p}^2 + \nu D k_\pm^2 - \omega_\pm^2 - \mathrm{i}\nu\omega_\pm + \mathrm{i}k_\pm\overline{E}\right]^{-1}. \quad (12)$$

The density perturbation oscillating at the forced frequency in equation (12) are obtained under the quasi-static approximation and by neglecting the Doppler shift due to travelling space-charge waves. We also neglect the contribution of transition dipole moment in the analysis of modulational instability to study the effect of nonlinear current density due to diffusion of the charge carriers only. The diffusion-induced nonlinear current densities for the upper and lower side-bands may be expressed as

$$j_1(\omega_+, k_+) = -eD\frac{\partial n_1(\omega_+, k_+)}{\partial x}, \quad (13a)$$

$$j_1(\omega_-, k_-) = -eD\frac{\partial n_1(\omega_-, k_-)}{\partial x}\cdot \quad (13b)$$

In a centrosymmetric system, the four-wave parametric interaction involving the incident pump, the upper and lower side-band signals and the induced acousto-optic idler wave characterised by the cubic nonlinear susceptibility tensor effectively results in modulational instability of the pump. The cubic nonlinear optical polarisation at the modulated frequencies is defined as

$$\mathbf{P}_\mathrm{eff} = \varepsilon_0\chi_\mathrm{d}^{(3)}\mathbf{E}_0(\omega_0, k_0)\mathbf{E}_1(\omega_+, k_+)\mathbf{E}_1(\omega_-, k_-). \quad (14)$$

The induced polarisation $\mathbf{P}_\mathrm{d}$ is treated as time integral of the nonlinear current density $j_1(\omega_\pm, k_\pm)$. The effective polarisation has contributions from both the individual side bands and can be represented as

$$\mathbf{P}_\mathrm{eff}(\omega_\pm, k_\pm) = \mathbf{P}_\mathrm{d}(\omega_+, k_+) + \mathbf{P}_\mathrm{d}(\omega_-, k_-). \quad (15)$$

Thus the effective nonlinear susceptibility of the electrostrictive medium induced by the carrier diffusion in a four-wave mixing process can be obtained using equations (12–15) as

$$\chi_\mathrm{d}^{(3)} = \frac{-2\mathrm{i}D\nu n_0 e^2 k_\mathrm{a}^4\omega_0^2(\eta^2-1)^2}{2\rho m[\omega_\mathrm{a}^2 - k_\mathrm{a}^2\nu_\mathrm{a}^2 - 2\mathrm{i}\gamma\omega_\mathrm{a}](\omega_0^2-\omega_\mathrm{c}^2)^2} \times\left[\left(\delta^2+\nu^2-\frac{k^2\overline{E}^2}{\omega_0^2}\right)+\frac{2\mathrm{i}k\overline{E}\delta}{\omega_0}\right]^{-1} \quad (16)$$

in which

$$\delta = \overline{\varpi}_\mathrm{p} - \omega_0 + \frac{\nu D k^2}{\omega_0}\cdot$$

The effective nonlinear susceptibility [Eq. (16)] may be termed as a diffusion induced third-order susceptibility of the crystal. It characterises the steady-state optical response of the medium and governs the nonlinear wave propagation through the medium due to diffusion of the charge carrier in presence of a transverse magnetostatic field. Hence the process may be termed as diffusion-induced modulational interaction. For nondispersive acoustic mode *i.e.* for $\omega_\mathrm{a} = k_\mathrm{a}\nu_\mathrm{a}$ rationalisation of equation (16) yields

$$\left[\chi_\mathrm{d}^{(3)}\right]_\mathrm{r} = \frac{D\nu n_0 e^2 k_\mathrm{a}^4\omega_0\left(\eta^2-1\right)^2}{\rho m\gamma\omega_\mathrm{a}(\omega_0^2-\omega_\mathrm{c}^2)^2} \times\frac{k\overline{E}\delta}{\left[\left(\delta^2+\nu^2-\frac{k^2\overline{E}^2}{\omega_0^2}\right)^2+\frac{4k^2\overline{E}^2\delta^2}{\omega_0^2}\right]}, \quad (17a)$$

$$\left[\chi_\mathrm{d}^{(3)}\right]_\mathrm{i} = \frac{D\nu n_0 e^2 k_\mathrm{a}^4\omega_0\left(\eta^2-1\right)^2}{\rho m\gamma\omega_\mathrm{a}(\omega_0^2-\omega_\mathrm{c}^2)^2} \times\frac{\left(\delta^2+\nu^2-\frac{k^2\overline{E}^2}{\omega_0^2}\right)}{\left[\left(\delta^2+\nu^2-\frac{k^2\overline{E}^2}{\omega_0^2}\right)^2+\frac{4k^2\overline{E}^2\delta^2}{\omega_0^2}\right]}\cdot \quad (17b)$$

Equation (17) can be employed to obtain the steady state gain *via* $[\chi_\mathrm{d}^{(3)}]_\mathrm{i}$ as well as the dispersive characteristics *via* $[\chi_\mathrm{d}^{(3)}]_\mathrm{r}$ of the modulated waves. It can be observed from equation (17a) that there is an intensity dependent refractive index leading to the possibility of a focusing or defocusing effect of the propagating beam. Equation (17a) reveals the negative dispersive characteristics of the dissipative medium at $\omega_\mathrm{c} > \omega_0$. As $[\chi_\mathrm{d}^{(3)}]_\mathrm{i}$ becomes negative one may expect more effective self focusing of the modulated signal for normal dispersion characteristics. Hence the application of magnetostatic field adds new dimensions to this interaction process. However we cannot increase the value of magnetostatic field indefinitely as with $\omega_\mathrm{c} \gg \omega_0$, cyclotron absorption phenomenon may dominate the instability process.

In order to explore the possibility of diffusion-induced modulational amplification in a centrosymmetric semiconductor, we employ the relation

$$\alpha_\mathrm{e} = \frac{k}{2\varepsilon_1}\left[\chi_\mathrm{d}^{(3)}\right]_\mathrm{i}\left|E_0\right|^2, \quad (18)$$

where α_e is the nonlinear absorption coefficient. The nonlinear steady state growth of the modulated signal is possible only if α_e, obtainable from equation (18), is negative. Thus from equations (17b, 18) we can infer that in the present case, in order to attain a growth of the modulated signal, $[\chi_\mathrm{d}^{(3)}]_\mathrm{i}$ should be negative. Thus the condition for achieving a positive growth rate is as follows:

$$k^2\overline{E}^2 > \omega_0^2\left(\delta^2+\nu^2\right). \quad (19)$$

Thus it is evident from above discussion that not only the presence of particle diffusion in an externally imposed magnetic field is an absolute necessity to induce instability but also the value of applied pump intensity must be greater than the threshold value defined by equation (19).

In order to determine the threshold value of the pump amplitude required for the onset of the modulational amplification, we set $k^2\overline{E}^2 = \omega_0^2(\delta^2 + \nu^2)$ and obtain

$$E_{0\mathrm{th}} = \frac{m}{ek}\sqrt{(\delta^2+\nu^2)}\frac{(\omega_0^2 - \omega_c^2)}{\omega_0} \cdot \qquad (20)$$

It can be observed from equation (20) that the transverse modulational instability of the signal wave has a nonzero intensity threshold, ever in the absence of collisional damping. The threshold field $E_{0\mathrm{th}}$ is found to have a complex characteristics and is strongly dependent on the externally applied magnetic field.

A detailed investigation about the nature of the steady state gain factor reveals that an appreciable amplification of the modulated signal ($g = -\alpha_\mathrm{e}$) is obtainable only under the condition of non-dispersion *i.e.* as $\omega_\mathrm{a} \to k_\mathrm{a}\nu_\mathrm{a}$. The above formulation reveals that the presence of a magnetostatic field enhances the growth rate of the modulated signal. It is found that the growth rate of the signal is independent of its frequency and instead, it depends on the frequency of the pump and that of the acoustic-wave; a fact in agreement with the experimental results (24). It is also found to be influenced by the carrier concentration n_0.

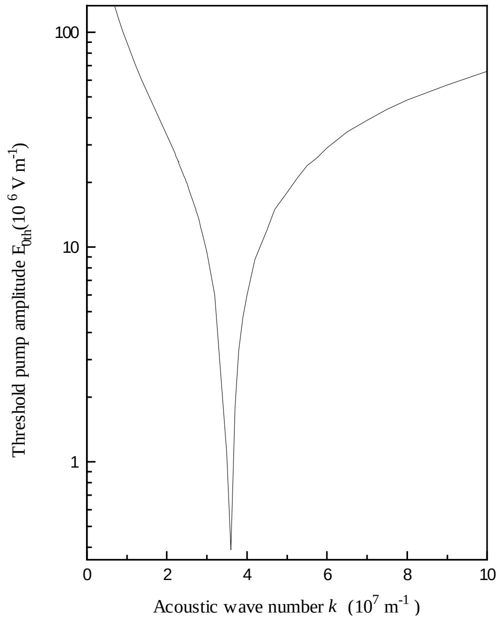

Fig. 1. Dependence of threshold pump amplitude $E_{0\mathrm{th}}$ on acoustic wave number k for $n_0 = 10^{24}$ m^{-3}, $\omega_\mathrm{a} = 10^{12}$ s^{-1}, $\nu = 4 \times 10^3$ m s^{-1}, $\omega_0 = 1.78 \times 10^{14}$ s^{-1}, $\omega_\mathrm{c} = 0.01\omega_0$.

3 Results and discussion

The diffusion induced modulational amplification of the copropagating waves in the electrostrictive Kerr medium is due to the linear dispersion effects in combination with the nonlinear processes. The amplification of the modulated electromagnetic wave is critically dependent on the coupling of the electron-plasma wave and the generated acoustic wave. Thus the amplification process can be controlled by the carrier density of the medium that governs the effective plasma frequency in the presence of the intense pump beam and the diffusion of charge carrier in the medium. The amplification is expected to be higher in the presence of a strong electron plasma wave that enhances the coupling between interacting waves. The amplification of the modulated wave is maximum for the most efficient coupling of the side band signal. Thus any process that reduces the phase mismatch will consequently enhance the modulational amplification process. The presence of strong acoustic wave in the system as an "idler" serves as an effective mean to reduce the phase mismatch between interacting waves.

An externally generated acoustic wave can also be subjected into the system to enhance the modulational amplification process as it would improve the grating strength. The presence of an external acoustic wave is found to add coherently to the induced acoustic wave in case of a Bragg diffracted single side band Stokes component (11). However, this additional sound wave will modified the Stokes-antiStokes coupling parameter and the space-charge field also needs to be adjusted to be accounted for in the present theory.

The analytical investigations for the possibility of transverse modulational instability and the consequent amplification of modulated waves resulting from the transfer of modulation from the pump wave to the modulated wave are dealt with in the preceding section. The analytical results obtained are applied to a centrosymmetric n-type III-V semiconductor being irradiated by 10.6 μm pulsed CO_2 laser at 77 K. The physical parameters considered for the analysis are: $m = 0.0145m_0$ (m_0 being the free electron mass), $\rho = 5.8 \times 10^3$ kg m^{-3}, $\nu = 3\times10^{11}$ s^{-1}, $\varepsilon_1 = 15.8$, $n_0 = 10^{24}$ m^{-3}, $\omega_\mathrm{a} = 10^{12}$ s^{-1}, $\nu_\mathrm{a} = 4 \times 10^3$ m s^{-1}, $\omega_0 = 1.78 \times 10^{14}$ s^{-1}.

The numerical estimations dealing with the external parameters influencing the threshold field required for the onset of modulational amplification process are plotted in Figures 1 and 2. It can be observed from Figure 1 that for smaller magnitudes of k_a (such that $\frac{\omega_0(\overline{\omega}_\mathrm{p}-\omega_0)}{\nu D} \gg k_\mathrm{a}^2$), $E_{0\mathrm{th}}$ decreases with k_a as k_a^{-1}, and at $\frac{\omega_0(\overline{\omega}_\mathrm{p}-\omega_0)}{\nu D} \approx k_\mathrm{a}^2$, $E_{0\mathrm{th}}$ is found to be minimum and further when $\frac{\omega_0(\overline{\omega}_\mathrm{p}-\omega_0)}{\nu D} < k_\mathrm{a}^2$ then $E_{0\mathrm{th}}$ shows a steep increment. Figure 2 shows the dependence of $E_{0\mathrm{th}}$ on the external dc magnetic field B_s (in terms of cyclotron frequency ω_c). It is found that the threshold field required for inciting the modulational amplification is much less at lower value of magnetic field. $E_{0\mathrm{th}}$ is found to increase till the magnetic field approaches 1 T (corresponding to a cyclotron frequency nearly equal to 1.78×10^{13} s^{-1}). However for $\omega_\mathrm{c} > 2 \times 10^{13}$ s^{-1} one encounters a drop in the value of the required threshold field at $k_\mathrm{a} = 2.5\times10^8$ m^{-1}. The occurrence of this maxima may be attributed to the dependence of $E_{0\mathrm{th}}$ on a factor $f(\omega_\mathrm{c}) = (\delta^2+\nu^2)^{\frac{1}{2}}(\omega_0^2-\omega_\mathrm{c}^2)$ as evident from equation (20). Thus the presence an external transverse dc magnetic field for which $\omega_\mathrm{c} > 2\times10^{13}$ s^{-1}, effectively reduces the threshold field. This behaviour may be attributed to the presence of effective Hall field induced by applied transverse dc

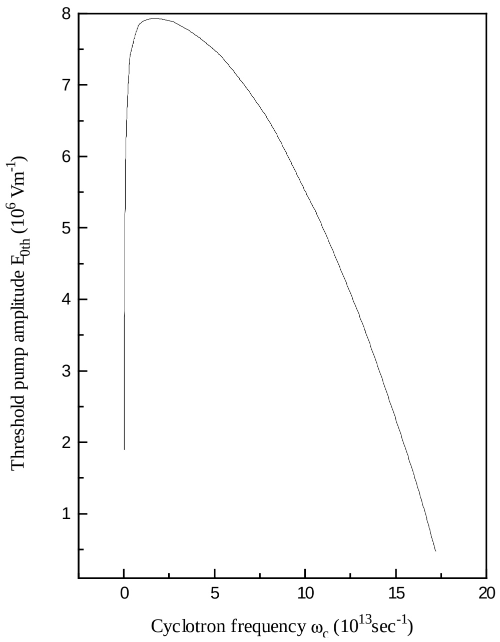

Fig. 2. Dependence of threshold pump amplitude $E_{0\text{th}}$ on magnetic field *via* cyclotron frequency ω_c for the parameters as in Figure 1, when $k = 2.5 \times 10^8$ m^{-1}.

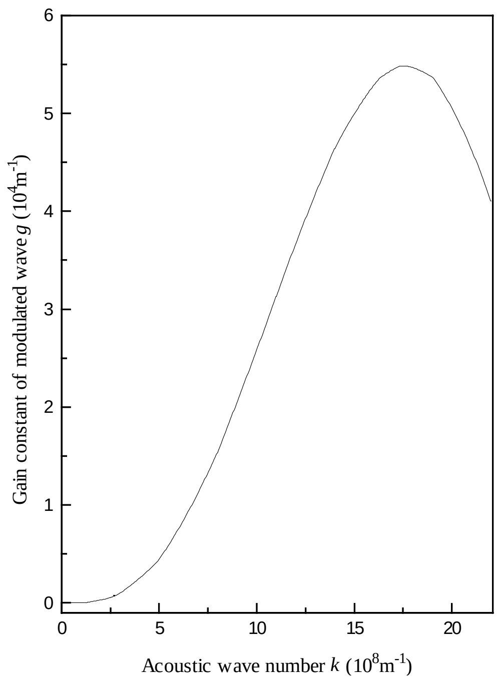

Fig. 3. Dependence of gain constant of modulated wave on wave number k when $\omega_c = 0.001\omega_0$.

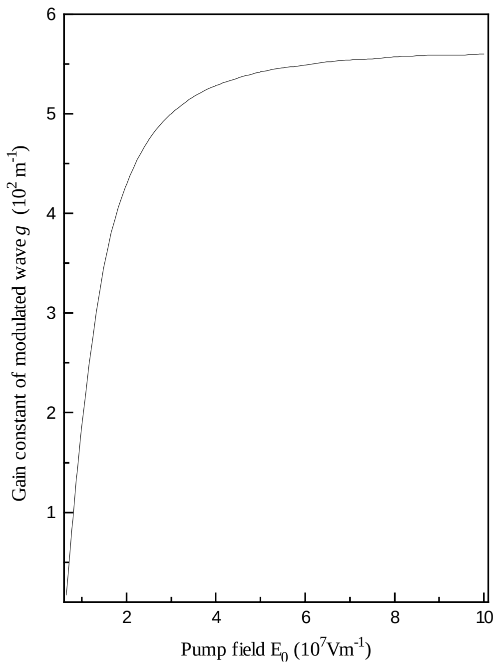

Fig. 4. Variation of gain constant of modulated wave with pump field E_0 for the parameters as in Figure 1. Here $k = 2.5 \times 10^8$ m^{-1}.

magnetic field corresponding to cyclotron frequency more than 2×10^{13} s^{-1}.

Figure 3 depicts the variation of the modulated growth rate g with respect to wave number k. The gain constant g has the usual $g \propto [ak^2(b|\mathbf{E}_0|^2 - ak^2)]^{\frac{1}{2}}$ dependence on k characterising a parametric four-wave coupling process [25]. This results in the gain of the modulationally unstable propagating signal g increases initially with k and attains a maximum around $\omega_a \approx k\nu_a$, *i.e.* the nondispersive acoustic mode. However, in the negative GVD regime (when $\omega_a < k\nu_a$); g drops sharply due to the focusing of the beam as evidenced by the positive dispersive characteristics of the AO susceptibility tensor [Eq. (16)].

Figures 4 to 6 display the numerical estimations of equation (18) at electric fields higher than the threshold value. These estimations are plotted for a nondispersive acoustic mode with $\omega_a \approx k\nu_a$ (*i.e.* with $k_a = 2.5\times10^8$ m^{-1}, $\nu_a = 4 \times 10^3$ m s^{-1} and $\omega_a = 10^{12}$ s^{-1}) in the heavily doped regime ($n_0 \approx 10^{24}$ m^{-3}) g increases with E_0 after overcoming the attenuation below the threshold field and exhibits a nearly independent behaviour due to modulation of effective acousto-optic modulation susceptibility which is induced by the space-charge field. Here g is now independent of applied electric field as the diffusive forces are balanced by the enhanced dielectric relaxation frequency $\overline{\omega}_R$ and electron plasma frequency ω_p. In the heavily doped regime due to the enhanced plasma frequency $\omega_p \to \omega_0$ the density of the space-charge wave is also increased resulting in a reverse transfer of energy from the acousto-optic field to the material waves in the resonant regime resulting in an attenuation of modulated wave. However, as the pump field increases further, it becomes strong enough to derive the space-charge wave overcoming dragging effects due to the diffusive forces and therefore exhibits an exponential growth. The variation of this modulated mode gain is analogous to the output characteristics of doped semiconductor diode arising due to optical instabilities [26]. The variation of the growth rate with dc magnetic field B_s (in terms of ω_c) has been

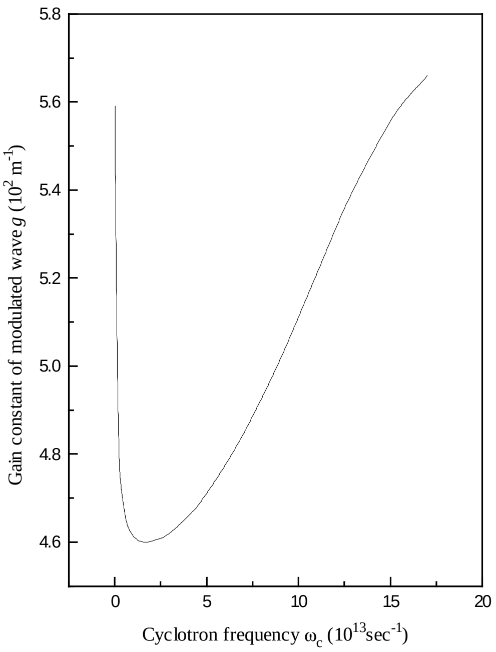

Fig. 5. Variation of gain constant of modulated wave on magnetic field *via* cyclotron frequency ω_c for the parameters as in Figure 1, when $k = 2.5 \times 10^8$ m^{-1}.

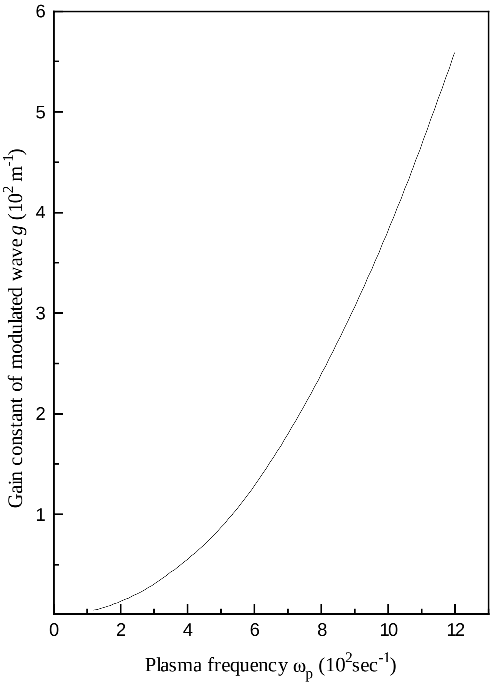

Fig. 6. Variation of gain constant of acoustic wave with carrier concentration *via* ω_p for the parameters as in Figures 1 and 2.

depicted in Figure 5 taking k as a parameter. It is found from this curve that g decreases sharply with the increase in ω_c and attains a minimum value at $\omega_c \approx \nu$. In this part of the curve the magnetic field has been trying to overcome the frictional losses and as soon as it overcomes ($\omega_c > \nu$) the frictional losses due to increase in the Hall drift energy the gain increases with the increase in the applied dc magnetic field. But we cannot increase the value of ω_c indefinitely because after a certain value cyclotron absorption becomes important and one has to restructure the present theory accordingly.

In Figure 6 we have plotted the growth rate of the signal as a function of electron density n_0, for a dispersionless regime of the low frequency acoustic mode. It is found that the growth rate of the transversely modulated wave increases with a rise in electron density of the medium. The nature of the curve is similar to the conclusion arrived at by Salimullah and Singh [27] who considered the modulational interaction of an extraordinary mode subjected to perturbation parallel to magnetic field. Hence higher amplification of the waves can be attained by increasing the carrier concentration of the medium by n-type doping in the crystal. However, the doping should not exceeds the limit for which the plasma frequency ω_p exceeds the input pump frequency ω_0, because in the regime where $\omega_p > \omega_0$ the electromagnetic pump wave will be reflected back by the intervening medium. It may be thereby concluded that heavily doped semiconductors are the most appropriate hosts for diffusion-induced modulational instability processes.

The preceding analysis has been performed for III-V semiconductors like n-type InSb with electron density approaching critical density (*i.e.* carrier densities for which the corresponding electron plasma frequency is comparable to the incident pump frequency $\omega_0 \approx \omega_p$). Carrier densities of such high magnitudes are quite relevant to semiconductors of the III-V group [28] and have been extensively employed by several workers to study the various characteristics [17,22,27].

4 Conclusions

The present paper deals with the analytical investigation of diffusion-induced acousto-optic modulation of an intense electromagnetic beam due to acoustoelectric effect in nearly cubic crystals such as III-V semiconductor plasmas. Owing to the strain dependent nature of the medium the intense laser field generates an acoustic wave packet in the acousto-electric domain of the active plasma medium. This leads to an enhanced electron-phonon interaction which gives rise to a strong space-charge field associated with the lattice vibrations of the medium resulting a modulation of the input light beam. The following important conclusions can be drawn on the basis of the above study.

(i) A maximum gain of the diffusion-induced modulated pump beam can be achieved by operating the device in the dispersionless acoustic wave regime within a heavily doped plasma medium.

(ii) An efficient modulation of the pump can be achieved by maintaining the acoustic field considerably lower than the pump field and by controlling the pump frequency, dielectric relaxation frequency, diffusion coefficient and acoustic wave number such that

$$\left[\frac{\omega_0(\overline{\omega}_R - \omega_0)}{\nu D}\right] < k_a.$$

(iii) The amplification of the signal can be obtained in the acoustoelectric domain which can be controlled by selectively doping the medium as the modulation of the pump beam is considerably dependent upon the carrier density of the medium.
(iv) The presence of an external dc magnetic field is favourable for the onset of the diffusion-induced modulational amplification of the modulated waves in heavily doped regimes. For a magnetic field of magnitude more than 1 T, the parameter not only reduces the threshold field required for the phenomena but also enhances the gain constant at the output and thus play a very significant role in the process.

The present theory thereby probably first time establishes the possibility of diffusion-induced modulational interactions in centrosymmetric semiconductor plasma medium. It also provides an insight into developing potentially useful acousto-optic modulators by incorporating the material characteristics of the diffusive plasma medium.

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